CMEC
California Mathematics Education Collaborative

Instructional Quality Commission
1430 N Street, Room 3207
Sacramento, CA 95814

To the Members of the Instructional Quality Commission,

The California Mathematics Education Collaborative (CMEC) is an intersegmental group of educators and professionals working to improve mathematics education from the transitional kindergarten through postsecondary (TK-16) levels, with a focus on closing gaps at key transition points along the pipeline to college-level mathematics.

CMEC initially formed in response to a national call by the Conference Board of the Mathematical Sciences (CBMS) to assemble state teams for the CBMS forum in May 2019, *High School to College Mathematics Pathways: Preparing Students for the Future*. California's original CBMS team has since evolved into a 31-member think tank of mathematics educator leaders representing organizations from across California, including state agencies, school districts, county offices of education, community colleges, public and private higher education institutions, nonprofits and advocacy organizations. For a complete member list, see here.

CMEC's driving vision is to empower all California students in relevant mathematics learning for their educational and career aspirations. To achieve this vision, our collaborative work focuses on three areas: math courses and student success, professional learning for educators, and educational policy and alliances. Because the guidance offered in the California Mathematics Framework has significant implications for our work, our CMEC think tank underwent a collaborative review of the current Framework Revision.

This letter represents our collective feedback, and is designed to uplift and affirm components of the Framework we agree with as well as offer specific suggestions for areas needing clarity. Our comments aim to leverage the clarity of purpose regarding how to achieve equitable access to advanced mathematics in California -- a clarity of purpose we've been able to achieve through working together as an interinstitutional, intersegmental group of mathematics educators over time. Our affirmations and suggestions for refinement align with CMEC's driving purpose and the related objectives of our three focal areas; thus, much of our feedback focuses on Chapters 5, 8, 9, 10 and the Appendix.

In general, we affirm that there are many more options than early over-acceleration towards AP Calculus in high school. While students should have opportunities to accelerate to take AP math coursework, we believe that if this is the only pathway of value, it sets up a competitive zero sum stance around "high tracks" and "low tracks." As an intersegmental body that represents TK-16 mathematics educators and partner organizations, we applaud the authors of the Framework for addressing this core equity issue. Chapter 2, lines 80 - 82, state: "This framework also offers ideas for teaching in ways that promote racial justice and create space for students with a wide range of social identities to feel a sense of belonging as they are to the mathematics community." We believe this approach to mathematics is core to changing the status quo of inequitable access to advanced mathematics across California, and will support all students in relevant mathematics learning for their educational and career aspirations while creating communication and trust between families, schools, and higher education institutions.

More specific comments, by chapter, page and line item, are included in Appendix A, attached below.

Thank you for your time and consideration,
The Members* of the California Mathematics Education Collaborative
Names and organizational affiliations of the intersegmental signatories to this letter

Appendix A

CMEC Affirmations and Modifications

CA Mathematics Framework: Chapters 5, 8, 9, 10 and the Appendix

CHAPTER 5 FEEDBACK

Overall, we celebrate the chapter's intention to reimagine the mathematics curriculum and center mathematical learning as the development of concepts and skills employed by students to wonder about and explore their world and their ideas. We hope the suggestions for modification included in our feedback below help strengthen and clarify this intention.

Page 5

L***ines 76-79***: We affirm the idea of data science as "the process of uncovering the stories hidden within data…" There is much public discourse about this idea, and we want to confirm the importance and relevance of data and statistics across many fields.

Lines 86-89: We believe, however, that the use of the term data science is questionable and that the statements on these lines warrant further clarity. For example, "Data science highlights the expansion in computing and visualization tools that have made many more techniques available for finding meaning in data—often relying on innovative visualizations of complex data that enable major features to be identified and explored further." **We recommend using the terms data fluency and data literacy rather than the term data science.** The area of Data Science is an existing academic field of research that relies on Calculus-based Statistics, Linear Algebra, and advanced Computer Science. It is not worth creating an argument over terms by insisting on using the same term to refer to something completely different than an established academic and commercial area of specialization.

Lines 89-94: We appreciate the distinguishing language between data science and statistics offered on these lines; we find this distinction helpful.

Page 10

Line 214: **We affirm columns 1 and 2 of the table** describing the big ideas in Statistics and Data Literacy for TK-5 and 6-8.

Line 214: We are concerned about **column 3 of the table** because it is not clear how teachers will achieve the additional knowledge needed to teach these big ideas. We recommend specifically addressing the related professional development needs such a shift in instruction will require, here and again in chapter 8.

Pages 28-30 & 35

We affirm the use of data collection and analysis in K-5 to contextualize the learning of whole numbers and later fractions, decimals and percents. This is illustrated by lines 515 and 570 below:

Line 515: "Categorical (non-numerical, or qualitative) data and measurement or quantitative data. For instance, consider a set of colored blocks in the classroom. 'Color' is a categorical or qualitative variable that students could observe about each block. 'This block is 15 centimeters long' is a measurement data point. The standards develop categorical data in kindergarten through grade three and measurement data beginning in grade two."

Line 570: "In third grade, contexts for questions to investigate using data should expand to include volume and mass measurement (grams, kilograms, and liters, but not compound units such as cm3) in addition to the length, time, and money contexts from earlier grades (3.MD.A.2). Time measurements are refined to the nearest minute (3.MD.A.1) and length now includes half- and quarter-inches (3.MD.B.4). Beginning ideas of area give another possible context, limited here to areas that can be covered by a whole number of unit squares (3.MD.C.5, 3.MD.C.6)."

Page 40

We affirm the study of data fluency and literacy and statistical thinking in Grades 6-8. For example:

Line 840: "Middle school includes a big expansion in important ideas. The big ideas of data science include:

> *Data in the world: exploration, interpretation, decision making, ethics; *Variability: Describing, displaying, and comparing;
> *Sampling to understand a population: randomness, bias, how many?;
> *Are they related? Multivariate thinking;
> *What are the chances? Probability as the basis for data-based claims."

Pages 62 & 63

We are concerned about the lack of clarity about the parts of the CA CCSS-M that might be de-emphasized while recommending that educators place greater emphasis on statistical and data fluency and literacy. For example:

Line 1465: "When using a sample mean or proportion to estimate a population mean or proportion, students use simulation models to estimate a margin of error, instead of formulaic calculations."

This is a very heavy lift. Most high school statistics teachers will need support to understand the connections between the models and the thing being modeled. E.g., why can we use a computer program to model the M&M experiment?

We suggest revising to read:

*"*When using a sample mean or proportion to estimate a population mean or proportion, students use simulation models, instead of formulaic calculations. Teachers will need support in making the connection between observations and expectations based on probabilities."

We recommend adding general language to this section that specifically states its' intent to ***stitch** data and statistics* throughout students' mathematics experiences while being more specific about what parts of the CA CCSS-M will be de-emphasized. Recommendations IIB and IIC of the final report of Quantitative Reasoning Task Force (QTRF), can guide this; recommendation IIC, in particular, defines the foundational quantitative reasoning required by the California State University and Community College systems.

Page 64
We affirm:

Line 1516: "Important discussions for students to engage in when working with existing data sets include: *Prior to exploring: Do we expect any of these variables to be associated? Why? *Might the association we see just be a result of the way in which the data was collected, rather than truly reflective of the population? What features of the data collection might make conclusions suspect, and what features might give confidence? Note that a large sample size is not enough to have confidence in conclusions. *Can we think of possible explanations for the association(s) we see? Can we think of ways we could decide which explanations might be accurate?"

Pages 80-81

We are concerned about the expectations placed on teachers in an environment where professional development and lesson study are not integrated into teachers' professional work day. For example, line 1977 notes that "Effective Data Science courses consider how to help students with the following" and then goes on to list 11 learning outcomes for students that are not attainable for the majority of current mathematics teachers:

Lines 1978-1981 & 1983-1984: The Framework should suggest models for teacher preparation and professional development that would support teachers in the social science or science disciplines required by these lines. There should be clarity around how the suggested content significantly differs from current practice and training.

Lines 1982 & 1992-1993: We have similar recommendations for supporting novice and more veteran teachers as they take on topics in statistics at a much deeper level (e.g. discussing what variability means in many contexts rather than focusing on procedures).

Lines 1992-1995: We have similar concerns about supporting novice and more veteran teachers on the programming and data ethics expectations.

Lines 1996-1999: These fall beyond the comfort zone of most math novice and more veteran teachers.

We recommend using very clear language in this chapter, and then again in chapter 10, that 1) acknowledges the cultural, instructional and knowledge shifts that will need to occur in order to meet the expectations for teaching outlined in sample courses such as these; and 2) clearly states that professional learning to facilitate these shifts should/must be integrated into teachers' professional work day using lesson study and similar approaches. If that work is done, we expect that the Framework may need to offer more approachable expectations for student learning, such as those found in the work of Phil Daro or the QRTF Report.

CHAPTER 8 FEEDBACK

Overall, we celebrate the chapter's attention to creating new high school options for students to provide each and every California student with access to the power of mathematics in a world that is increasingly quantitative. Our affirmations and suggested refinements below *aim to ensure these new options can prepare all students for their post-secondary career goals and aspirations.*

Page 7

Some parts of Chapter 8 lack clarity as to the parts of the CA CCSS-M that might be de-emphasized given the greater emphasis on statistical and data fluency and literacy called for in the Framework. For example:

Line 126: "Analyze the data: All graduates should be able to identify appropriate summaries (graphical displays, tables, summary statistics) for quantitative or categorical data, and to generate those summaries for some data sets using technology. For a relationship between two quantitative variables, they should be able to use appropriate technology to generate a correlation coefficient and a least-squares regression line, and then to interpret both in the context of the data. They understand that statistical claims about populations are based on probability."

In our collective experience, incoming students to the CSU system currently lack these skills due to an under-emphasis on data fluency and literacy in high school mathematics. We appreciate the Framework authors' intention to bring them to the forefront of students' learning experiences -- and simultaneously have concerns about the CA CCSS-M knowledge and skills that will be de-emphasized as a result. We again recommend that the authors refer to the QRTF Report, recommendations IIB and IIC, to clearly articulate how the Framework's emphasis on data literacy and fluency will prepare students' to meet the foundational quantitative reasoning requirements of entering CSU students.

Page 7

Much like the feedback we shared in Chapter 5, lines 1978-1999, we are again concerned about the expectations placed on high school teachers in an environment where professional development and lesson study are not integrated into teachers' professional work day. For example, currently most teachers lack the confidence with statistical and data manipulation concepts to teach what is proposed in lines 147-152; see below.

> *Lines 147-152:* "Interpret results: Graduates can interpret the results of their analysis in the context of the statistical investigative question. They can explain the meaning of population estimates or other results, and discuss possible sources of error such as missing data and imperfect data collection. They are

able to interpret margins of error and confidence intervals, demonstrating correct probabilistic understanding. They can communicate their results via writing, speaking, and visual representations."

We recommend changing the above to read:

Lines 147-152: "Interpret results: Graduates can interpret the results of their analysis in the context of the statistical investigative question. They can explain the meaning of population estimates or other results, and discuss in general terms possible sources of error such as missing data and imperfect data collection. They are introduced to the concepts of margins of error and confidence intervals graphically, practicing correct probabilistic understanding. They can communicate their results via writing, speaking, and visual representations."

Page 8
Lines 163-169: We affirm "Exploring Changing Quantities (CC 2): "Reading and writing with mathematics involves recognizing quantities in situations; translating relationships between them from natural language, visual, or other forms into mathematical forms (often equations, but also graphs, tables, and more); working with and moving between these mathematical forms to understand or answer questions about the relationships; and interpreting findings back in the original context. All students should develop this inclination and ability to a significant degree."

This is something we see students struggle with at the post-secondary level, particularly when the math is contextualized within their major (e.g. Business, Materials Science, etc.). Here if the K-8 Framework were successful at introducing variables with the language of data literacy, perhaps algebra and functions would seem more natural.

Page 9
Lines 203-205: We affirm "Much of the power of mathematics as a lens for understanding authentic contexts and problems lies in the fact that the same mathematics (when abstracted from the particular quantities in the current context) applies to such varied situations."

This is a critical realization: contextualization is essential and at the same time mathematics is powerful because one idea or concept can model many different situations. If students are to be fluent users of quantitative reasoning, their mathematical skills and practices must be integrated and connected, rather than being held as isolated memorized instances.

Page 10

Lines 209-216: We affirm "All high school graduates should be able to apply reasoning about linear, quadratic, and exponential functions across a variety of contexts, and interpret that abstract reasoning in the particular quantities of those contexts (SMP.2). Students should understand abstraction as a way to reason similarly across different contexts (SMP.8). For example, the contexts of population growth, interest-earning accounts, and radioactive decay were not designed to be applications of exponential functions; rather, exponential functions are noticed, described, defined, and studied because of the observed similarity in reasoning about these (and many more) contexts."

This is in line with the minimal mathematical proficiency needed to broadly access the education offered at the CSU and CA Community Colleges, and agrees with what the QRTF report (2016) called Foundational Quantitative Reasoning (in recommendation IIC on page 14).

Page 16

Line 397: We affirm: "Five Components of Equitable and Engaging Teaching. This framework's Chapter 2 (Teaching for Equity and Engagement) is structured around five components of equitable and engaging teaching, which are briefly revisited here. The components should inform high school instructional design as much as earlier grades. For much fuller discussions, refer to Chapter 2."

Line 412: We want to affirm item 3 "Teach toward Social Justice,"and underscore that doing this well will require professional support for teachers.

Pages 19-20

We propose an addition/change to this section:

> *Line 471:* In general the Framework does not connect CA CCSS-M topics to the Big Ideas, Content Connections and Drivers of Investigation. One way to connect topics to pedagogy would be by adding a column to the table on this page. Currently it has three columns: "Standards for Mathematical Practice - The 'How'; Content Connections - The 'What'; and Drivers of Investigation- 'The "Why.'"
>
> Between the "What" and the "Why," we recommend adding a "***Using/With/By"*** column which would show *how* the CA CCSS-M topics *fit into the* Standards for Mathematical Practice, Content Connections, and the Drivers of Investigation.

For example, the new table could look like the below or this could be included in an appendix:

Standards for Mathematical Practice - The 'How'	Content Connections - The 'What';	Using/With/By	Drivers of Investigation- 'The "Why."'
Students will…	while…	using...	in order to…
(SMP.1.) Make Sense of Problems and Persevere in Solving them	(CC1.) Communicating Stories with Data	Two-way tables and the concept of homogeneity	(DI1)Understand similarities and differences between two groups from a population.
(SMP.1.) Make Sense of Problems and Persevere in Solving them	(CC2.) Exploring Changing Quantities	Linear (exponential, quadratic...) models	(D12) Predict What Could Happen (in a particular situation as one of the quantities increases…)

Page 30

Line 747: We affirm the concept that "different conceptions and unfinished learning add value to classroom discussions when they can be made visible and used thoughtfully. Activities should be designed to elicit common mis- or alternative conceptions, not to avoid them. This requires that teachers work through tasks before using them in classes, in order to anticipate common responses and plan ways to value contributions and use them to build all students' understanding. The goal of mathematics class must be deeper understanding and more flexibility in using and connecting ideas—not quicker answer-getting (Daro, 2013)."

Surfacing mis- or alternative conceptions does add value to the classroom. Class time spent on connecting two correct approaches is critical if students are to have quick reliable recall and be able to transfer knowledge from one context to another. Furthermore, debugging as a class answers that are "not correct yet" helps disrupt hardwired bad habits and gets students in a class thinking much more than just immediately rushing to highlight the correct answer that came in the expected format.

What would be great here would be more scaffolding on how to do this because it requires time, skill, and legitimate confidence. There are re-engagement tools available in the public domain. For example, the Silicon Valley Mathematics Initiative (SVMI) partnered with the Dana Center to define the purpose of re-engagement lessons, provide videos of actual re-engagement lessons taught in classrooms and share the Tools for Teachers which uses actual student work and re-purposes the work in lesson plans to meet students where they are and move learning forward. These public domain resources are available at: https://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons

The Tools for Teachers are created from actual student work from MARS Performance Assessment Tasks. There are a wide variety of Toolkits for a range of grades/course and available to all at: https://www.insidemathematics.org/sites/default/files/materials/aaron%27s%20designs.pdf.

Page 35

Line 870: We are concerned with the depiction offered in figure 8.3, as the pathways seem to lock students into a path and may end up being troughs. **We recommend dropping the concept or idea of "*pathways*" and focusing instead on the "*options*"** depicted in the cloud image (Figure 8.3). For example, in choosing their 3rd and 4th math classes students could opt to go deep into one subject like statistics, data literacy or calculus. They could also try different options to help them make an informed choice in their post secondary experience (work, college or university), for example, by taking data literacy and then pre-calculus.

CHAPTER 9 FEEDBACK

Overall, we celebrate the chapter's attention to confront long standing inequities in how our students experience school math and how much access they have to the beauty and power of mathematics. We celebrate switching the focus from maintaining a system that has, historically, safely provided access and opportunity to a select few to one that seeks to provide access and opportunity to all.

Pages 6-8

Lines 135 -184: We affirm that this entire section locates the conversation about tracking within decades of research, and offers evidence to support the Framework's guidance away from tracking.

Pages 7-8

Lines 166-184: We affirm that tracking significantly increases inequality in learning and agree that, overall, students perform better in a non-tracked system. We appreciate the inclusion of Rui's 2009 meta-analysis. **We recommend that the authors quote directly from the reference:**

> *"The findings suggest that the detracking reform had appreciable effects on low-ability student achievement and no effects on average and high-ability student achievement. Therefore, detracking should be encouraged, especially in schools where the lower-track classes have been traditionally assigned fewer resources."*
>
> The ***above quote should replace*** **what is currently in the Framework on lines 167-171**: "students taught in non-tracked groups that offer a more ambitious curriculum tend to have higher achievement overall. This overall improvement is attributed to significant increases for low and middle achievers and no change for high achievers, who [the studies found typically] achieve equally well (and sometimes a bit better) in non-tracked systems."

Page 9
Lines 211-214: We affirm that teaching heterogeneous classes should be accompanied by high-quality professional development, and emphasize that whenever possible such professional development be integrated into the teacher's professional work day.

Page 10
Lines 248-251: We affirm the concept of opening pathways by eliminating low level classes and teaching more multidimensional mathematics to all students.

Page 15
Lines 350-7: We affirm that a substantial commitment on the part of districts is required to support teachers in implementing challenging and engaging instruction.

Page 16
Lines 365-6: We caution against the inclusion of "or with the support of computerized systems" as this can be misinterpreted and implemented as students engaging in solo mathematics. If and when technology is implemented as a tool to support conceptual understanding and also to support fluency, recommendations might point to dynamic modeling platforms such as Desmos or Tuva.

Page 21

Lines 503-537: We affirm the shift to more flexible versions of student grouping. We caution against the use of terms like “pathways” and instead champion the idea of “mathematics course options.”

Page 22

Lines 538-557: We affirm the inclusion of the Mathematics Placement Act. We recommend that the authors consider adding excerpts of the legislative declarations to clarify the purpose of the Mathematics Placement Act in providing equitable access to higher level mathematics for students of color, especially for students matriculating from K - 8 to high school districts:

> SB359 Section 1 (c) The most egregious examples of mathematics misplacement occur with successful pupils and, disproportionately, with successful pupils of color. (d) Mathematics misplacement has far-reaching impacts on a pupil’s confidence, general knowledge of mathematical concepts, and high school experience, and may also impact the college and career opportunities available to the pupil. (e) New research shows that it is less common for pupils of color, even high-achieving pupils of color, to reach calculus by grade 12 compared to their white and Asian peers.

CHAPTER 10 FEEDBACK

Overall, we celebrate the chapter’s attention to the teacher as a professional. We appreciate the particular emphasis on teacher collaboration and holding teaching as a group-worthy task. When teachers make commitments to one another about how they will strive towards equitable learning outcomes within a school or a department, their own efficacy and resilience will grow alongside increased efficacy and resilience of their students.

Page 6

Lines 122-124: We affirm the emphasis on aligned efforts to improve teaching and learning (Teachers and teacher leaders prepared …School, district, and county office administrators …Afterschool, early childhood, and other expanded learning opportunities …College and university faculty involved in …Community members and parents, guardians, and families…Formal and informal learning environments, including museums, libraries, science centers…). We especially affirm the inclusion of families and communities and informals in these efforts, and suggest adding even stronger language about the importance of leveraging families as authentic partners in the design and implementation of framework-aligned instruction and learning, particularly in TK-5 settings.

<u>Page 7</u>
Lines 172-175: **We affirm the ideas espoused in this section regarding the impact of teacher beliefs on students' ability to learn and overall mathematics identity.**

<u>Page 15</u>
Lines 368-389: We recommend refinements or additional clarity offered to the statement: *"The following considerations can provide support for prospective teachers of mathematics:"* If there is research based evidence that these considerations are effective, then that research should be referenced here.

<u>APPENDIX FEEDBACK</u>
In general, we appreciate the clarification about mathematical content progressions in the traditional and integrated pathways. However, we find the title "High School Pathways'' ineffective because in many school districts and in current public debate, the term "high school pathways" can be metonymic for necessary and often controversial conversations about course access, acceleration, and efforts to de-track mathematics. We recommend a more neutral title such as "The Arrangement of High School Content Standards." We also recommend that the authors more clearly describe the MIC or "Data Science" course progression options while changing "Data Science" to "Data Literacy" as suggested in our earlier feedback above.

<u>Page 26</u>
Lines 39 - 42: We appreciate the clarification that "'Traditional Pathway' refers to the organization of content, not to teaching practices. Although these courses are traditional in their content, they should be taught through active student engagement, as set out in Chapters 2 and 8, and whenever possible students should see and work on content that is conceptually integrated." While we do appreciate this clarification, we also wonder about the possibility within this Framework of **<u>renaming</u>** the "Traditional Pathway" with language that is potentially less problematic, such as *"Customary Course Progression."*

We affirm the clear distinctions made between algebra 1 and 2, for example in: "The system of polynomial functions, analogous to integers, is extended to the field of rational functions, which is analogous to rational numbers. Students explore the relationship between exponential functions and their inverses, the logarithmic functions. Trigonometric functions are extended to all real numbers, and their graphs and properties are studied. Finally, students' knowledge of statistics is extended to include understanding the normal distribution, and students are challenged to make inferences based on sampling, experiments, and observational studies."